PIMS day 1

Categories of boundary conditions

1d QFT that behaves topologically after taking $\overset{\text{BRST op}}{Q}$-cohomology

Hilbert space $\mathcal{H}$ is a dg vector space

some global symmetry whose charge mod 2 is fermion #

extension thereof

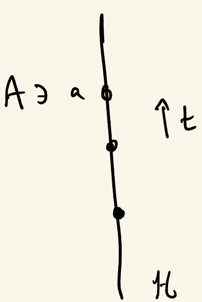
fermion #

i.e. full physical Hilbert space is $\mathbb{Z}$ (or $\mathbb{Z}_2$) graded

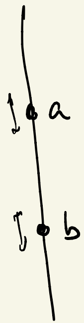
$\rightsquigarrow$ cohomological degree

w/ action of Q, $Q^2 = 0$, $|Q| = 1$

only care about cohomology $H^\bullet(\mathcal{H}, Q)$ i.e. $\mathcal{H}$ up to q-iso.



Local ops?



$$\begin{tikzcd}
 & & \text{dg algebra} \arrow[dl, "\sim"'] & \\
\text{assoc. algebra} & A_\infty \text{ algebra} \arrow[l, "\text{take}"'] & & E_1 \text{ algebra} \arrow[ll, "\sim"']
\end{tikzcd}$$

take name Q-coh.

IR

$\mu_2 = 0$ but higher ones may not be minimal

UV

↑ huge, all possible choices of insertion points

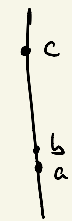
product

$a * b \neq$

$b * a$

not nec. commutative

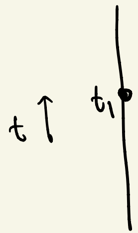
but



associative ✓

$c * (b * a) \simeq (c * b) * a$

Physical local ops in QM: Vec space $\mathrm{Ops}(t_1)$ for each time t_1



$a \in \mathrm{Ops}(t_1)$

Altogether $\mathrm{Ops} \downarrow \mathbb{R}_t$ flat sheaf (connection ∂_t)

$a(t) \in \mathrm{Ops}(t)$

In twisted context, operators come w/ descendants

for each $a \in \mathrm{Ops}$ there's a descendant $a^{(1)}$

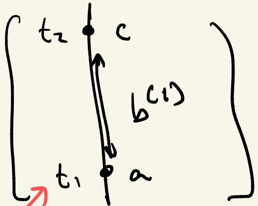
s.t. $\boxed{Q(a^{(1)}) = \underbrace{da}_{\partial_t a\, dt} - (Qa)^{(1)}}$

This ensures that Q-cohomology is t-invariant

If $Qa = 0$, $da = Q(a^{(1)})$ hence $= 0$ in cohomology.

Associativity at chain level: $Q b^{(1)} = db + (Qb)^{(1)}$

Consider

Q 

$=$ $-$ 

$+$ 

$+$ terms depending on Qa, Qb, Qc

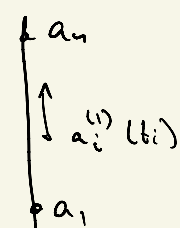
i.e. $c(t_2) \int_{t_1}^{t_2} b^{(1)} \; a(t_1)$ associator

but this might be Q-closed by accident.

If it is, it defines a local op in Q-cohomology

$= \mu_3(a, b, c)$

A_∞ algebra is constructed

by defining $\mu_n(a_1, \dots, a_n) :=$ 

$t_i < t_{i+1}$

2d | Topologically twisted QFT $\quad Q$

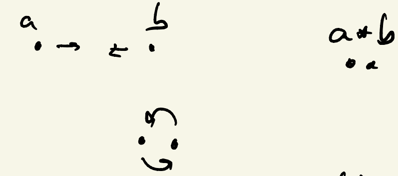
Local operators have the structure of an E_2 algebra

$$a^{(0)}(\vec{x}) \qquad a^{(1)} \qquad a^{(2)}$$

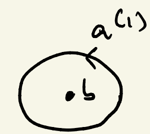
$$Q\, a^{(i)} = d\, a^{(i-1)} - \left(Q\, a^{(i-1)}\right)^{(1)}$$

In cohomology, always get

- product — graded-commutative!
- poisson bracket



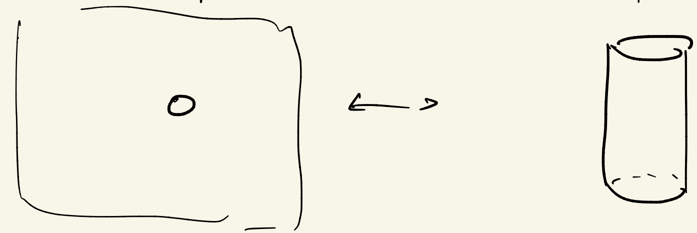
$\{a, b\} =$



$Q(\quad) = 0$ if $Qa = 0$ & $Qb = 0$

+ extra accidental higher operations (L_∞ operations)

State-operator corresp: $H^\bullet(\text{Local ops}, Q) \simeq H^\bullet(\text{Hilb space on } S^1, Q)$

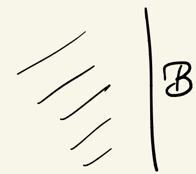


Boundary conditions of a 2d twisted QFT

form an A_∞-category which often has a derived model

objects are boundary conditions

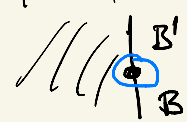
(preserving $\mathbb{Z}$-grading and Q)
($\mathbb{Z}_2$)



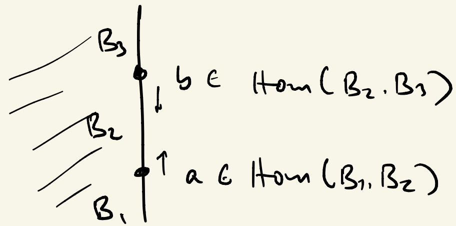
additive cat $\quad B \oplus B'$ makes sense $\quad$ (cf. $\oplus$ of Hilb spaces in QM)

morphisms $\quad \mathrm{Hom}(B, B')$ = space of local ops at a junction

Q-coh of ...

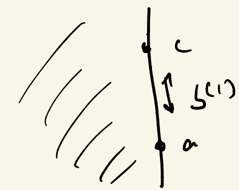


Naive coh of morphism spaces is associative



$\rightsquigarrow \quad b \circ a \in \mathrm{Hom}(B_1, B_3)$

Higher ops given by the same pictures



etc.

Theories of this type come from twists of 2d $\mathcal{N}=(2,2)$ QFT Kähler holomorphic

A & B σ-models labelled by $\mathcal{X}$, $W : \mathcal{X} \to \mathbb{C}$

	$\mathcal{X}$, $W = 0$	$\mathcal{X}$, $W \neq 0$	
A	Wrapped Fuk$(\mathcal{X})$	Fuk-Seidel$(\mathcal{X}, W)$	ie wrapped w/ stops
B	$D^b\,\mathrm{Coh}(\mathcal{X})$	$D^b\,\mathrm{MF}(\mathcal{X}, W)$.	

1d SUSY QM $(N=2)$ $[Q,\bar{Q}] = H$ $[Q,Q] := Q^2 = 0$

$[\bar{Q},\bar{Q}] = 0$

$$a^{(1)} := \bar{Q}(a)\,dt$$

$$\left(= [\bar{Q}, a]\right)$$

$$Q(a^{(1)}) = [Q,\bar{Q}](a) - \bar{Q}(Qa)$$

$$= \underbrace{H(a)}_{\partial_t a}dt - (Qa)^{(1)}.$$