Discrete Mathematical Modeling of Powder Bed 3D Printing Process

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3D Printers & Applications

Powder bed printers are machines in which a heat source melts the 2D cross section of the object into the bed of powdered material. Then another layer of powder is swept over and sintered onto the previous layer.

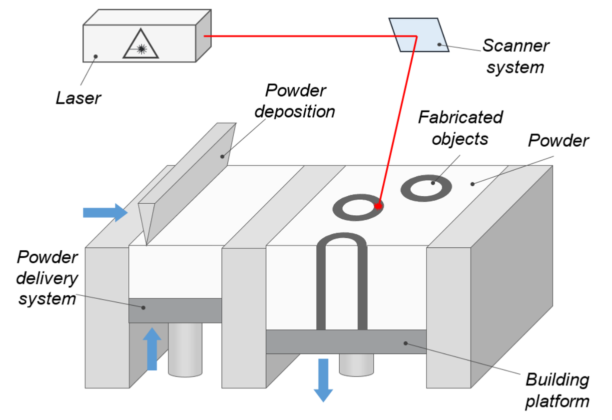
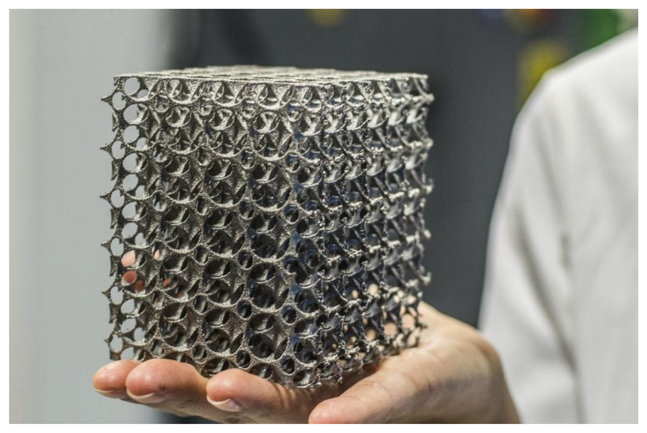


Diagram of Powder Bed Printer; from https://www.materialise.com/en/blog/metal-3D-printing-recoaters



Complex object produced using 3D printing; from http://engatech.com/metal-3d-printing/

3D printing is a cutting edge technology with multiple applications:

- Cooling channels which conform to the contours of the object for improved thermal dissipation.
- Building parts with complex geometry which would be difficult and expensive to manufacture with traditional methods.
- The option to build and test prototypes during the development phase.

Heat Source Model

- **Heat flux from laser:** $q_{i_{\text{laser}}} = Q\dfrac{r_i{}^3}{r_{\text{laser}}{}^3}$ where Q is the total power of the laser, r_i is the radius of the particle, r_{laser} is the radius of the laser.
- **Heat flux between two particles:** $q_{ij} = k_t\Big(T_j - T_i\Big)$ where k_t is the heat transfer coefficient, and T_i, T_j are the temperatures of particle i and j.
- **Total heat flux:** $q_i = q_{i_{\text{laser}}} + \sum_{j=1}^{N} q_{ij}$
- **Temperature update** for particle i: $T_i^{t+\Delta t} = T_i^t + \frac{q_i^t}{m_i C_p}\Delta t$ where T_i^t is the initial temperature, q_i^t is the total initial energy flux, m_i is the mass, and C_p is the specific heat capacity of particle i.
- If particle i and j are in **contact** and both above the **sintering temperture** a **bond** is formed between them.

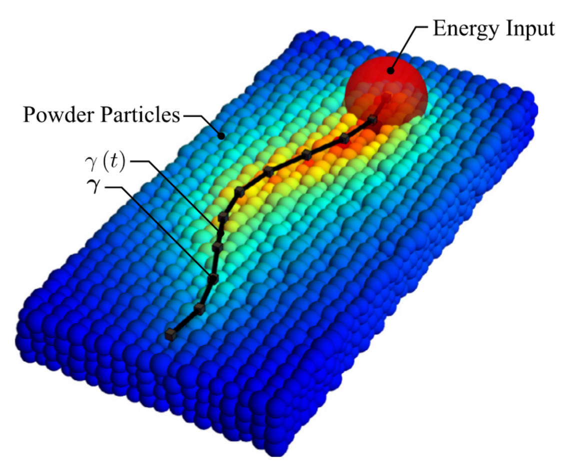


Diagram of model for heat source, γ is the laser path. [1]

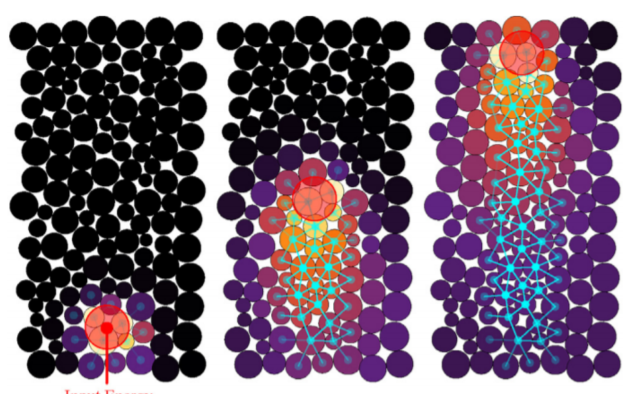
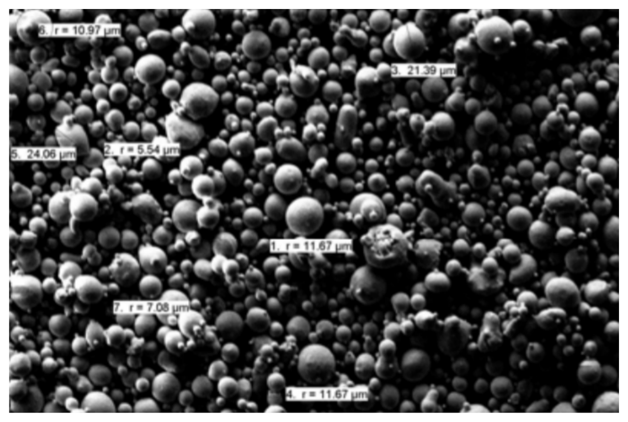


Diagram of heat source and bonding between particles. [1]

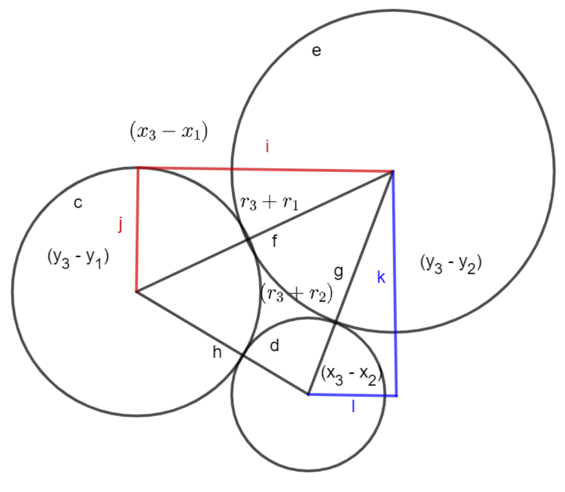
The Powder Bed Model

Our Algorithm:

- Every new sphere is randomly generated from chosen distribution.
- If two or more particles are in contact they can spawn a new sphere.
- The location of the new sphere is determined by solving a system of equations relating to the location and radii of the parent particles.



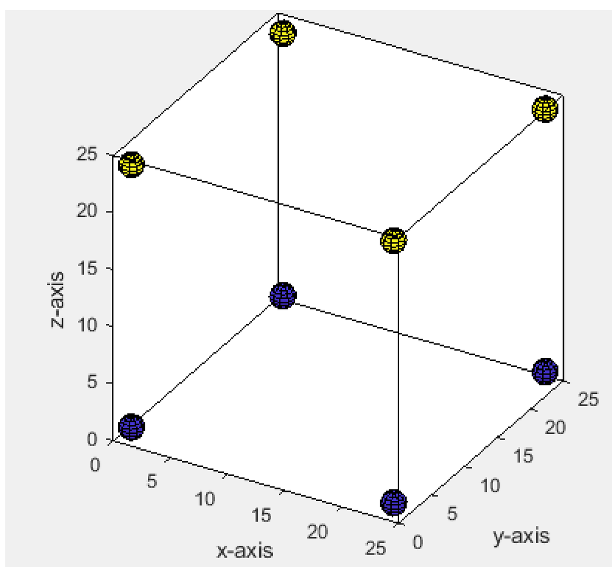
Close up of stainless steel type 316 powder. [2]



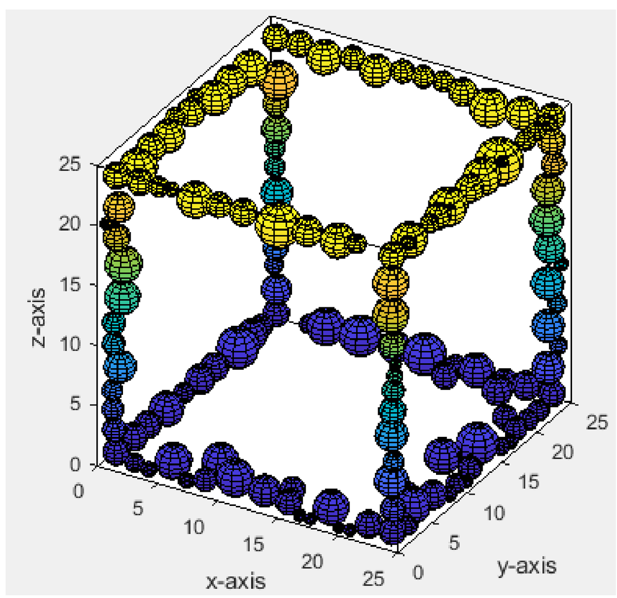
System of equations used to solve placement of new sphere.

The distribution of powder particle size was determined to be a **Weibull distribution:** $f(x;\lambda,k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{(-x/\lambda)^k}$ x is the diameter of the particle, $k = 3.55$ is the shape parameter, $\lambda = 31.4\ \mu m$ is the scale parameter, the average diameter is $28.3\ \mu m$ [2].

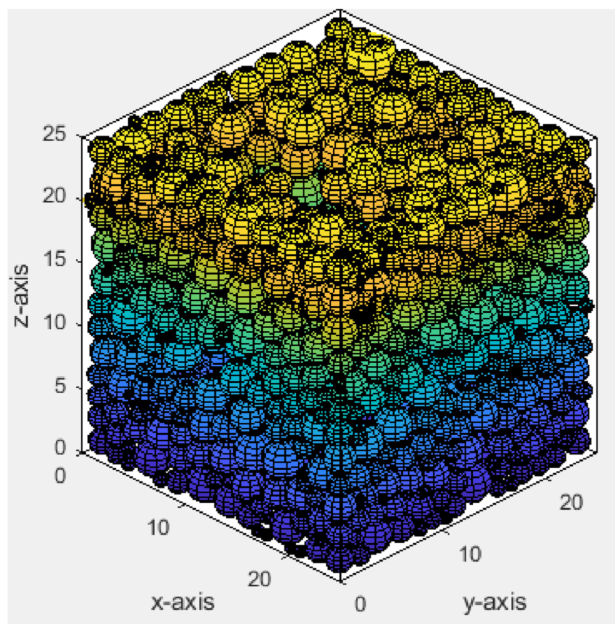
Packing of Random Spheres



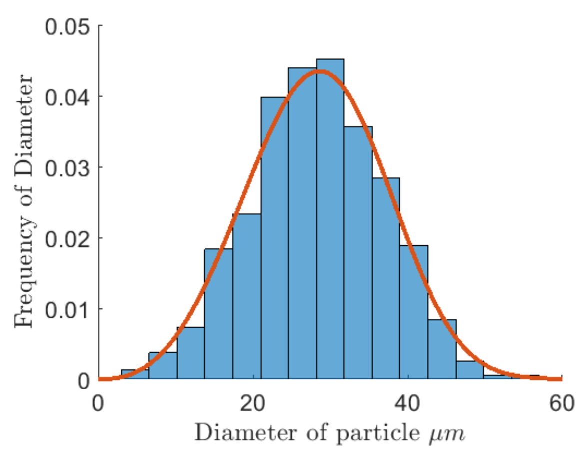
Spheres placed in the corners of cube.



Spheres placed in the corners of cube.



Spheres filling faces of cube.

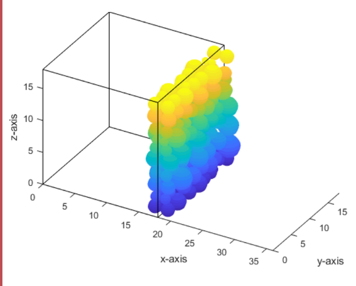


The data from the cube is represented by the bar graph, the Weibull distribution pdf is the red curve.

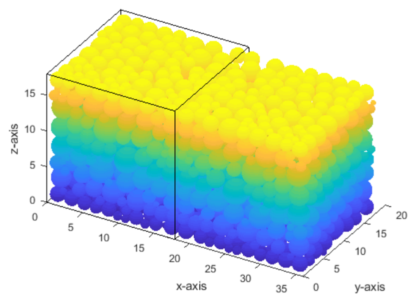
The distribution of spheres closely matches the probability density function from which the radii were randomly drawn.

Building the Powder Bed

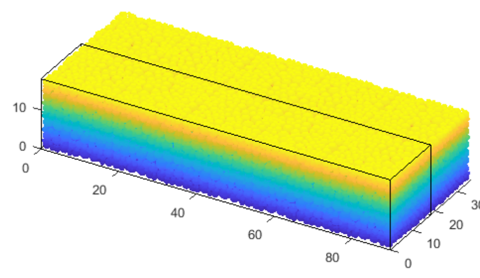
Once the cube is filled with spheres the bed is created by exploiting the symmetry of the cube. By reflecting the cube about its face it is possible to determine the contacts between two joined cubes.



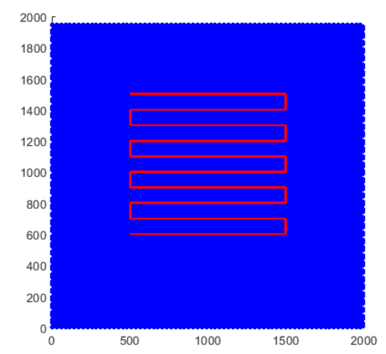
Cube contact.



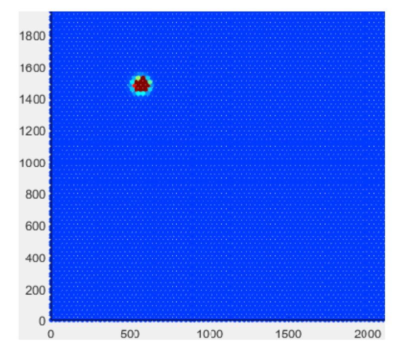
Two Unit Cubes.



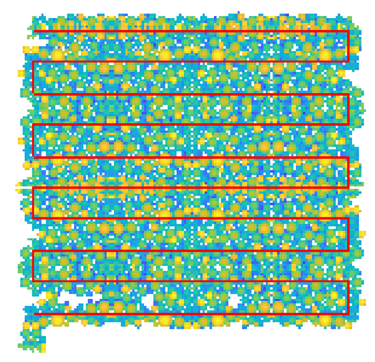
Simulated Print of a Square



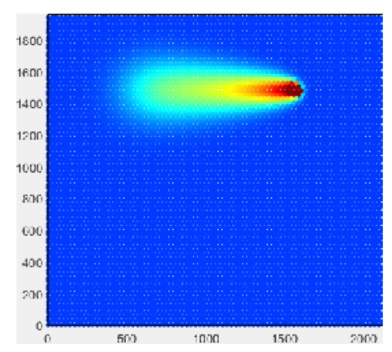
Path the laser will travel.



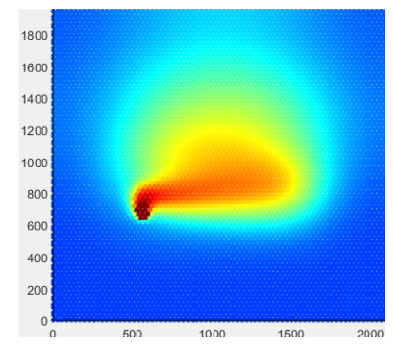
Initial state of the powder bed.



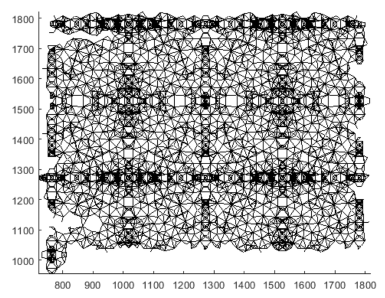
Particle bonded during print, with laser path superimposed.



First pass of the laser.



Final pass of the laser.



Bonds formed between particles.

Conclusions

Summary: An algorithm was developed to fill a cube with spheres of random radii. Symmetry of the cube was used to build a simulated powder bed. A discrete model of the 3D printing process was developed to study the affect of varying printing parameters.

Conclusions: The packing of particles will affect the final object. The path the laser takes will affect the internal bonding of the object. Residual heat from previous laser passes will affect the building process and must be taken into consideration.

References

[1] John G Michopoulos, Athanasios P Iliopoulos, John C Steuben, Andrew J Birnbaum, and Samuel G Lambrakos.
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Additive Manufacturing, 22:784–799, 2018.

[2] Adriaan B Spierings and Gideon Levy.
Comparison of density of stainless steel 316l parts produced with selective laser melting using different powder grades.
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